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NOTES.

THE IRVING HOPKINS FALLACY.

After my having so recently pointed out in THE AMERICAN MATHEMATICAL MONTHLY (Vol. III., pages 122-123) the fallacy of Professor G. C. Edwards of the University of California in his Elements of Geometry in treating parallells, and in *Science* (N. S. Vol. VI. page 491) the gross blunder made by Andrew W. Phillips and Irving Fisher, professors in Yale University, in their Elements of Geometry, could it have been supposed that so respectable a person as Irving Hopkins would deliberately have published the extended fallacy which has just appeared in THE AMERICAN MATHEMATICAL MONTHLY (Vol. IV., pages 251-255) under the ambitious title "Euclidean Geometry without Disputed Axioms"?

It is a simple *petitio principii.* The question is begged in his Proposition IV, which explicitly uses Euclid, III. 31. If any one will turn to III. 31 in any Euclid they will find it proved by Euclid, I. 32. But Euclid I. 32 is the famous angle-sum proposition, which since 1733 has been known to be equivalent to the parallel-postulate, the most disputed of all axioms.

In THE AMERICAN MATHEMATICAL MONTHLY's serial *Non-Euclidean Geometry*, (Vol. I., page 346) is given the Proposition : In any right-angled triangle the two acute angles remaining are taken together equal to one right angle, in the hypothesis of right angle ; greater than one right angle, in the hypothesis of obtuse angle ; but less in the hypothesis of acute angle. In other words, if the angle inscribed in a semicircle is right the geometry is Euclidean ; if obtuse, Riemannian ; if acute, Lobachevskian. GEORGE BRUCE HALSTED.

Austin, Texas.

There are several errors in Mr. Hopkin's paper on "Euclidean Geometry Without Disputed Axioms," but one is enough to which to call attention. In several places he uses Euclid III., 31, which depends upon I., 32, which depends upon I., 29, which depends upon Axiom 12 !

When will we cease trying to accomplish what the masters have found to be impossible ? BENJ. F. YANNEY.

Mt. Union College, Alliance, Ohio.

NOTE ON DR. LILLEY'S ARTICLE IN THE OCTOBER NUMBER.

There is one statement in Professor Lilley's article in the October number about which I wish to say a few words. Concerning the quotient 0. I define division thus : Having given the product of two factors, and one of the factors, to find the other factor.

Thus, the product of two factors$=12$,
One of the factors$=0$,
The other factor$=0$, (Lilley).

Hence $0\times0=12$. Do you believe it ?

Take this illustration—

$$
\begin{array}{l}
12 \div 3 = 4 \\
12 \div 2 = 6 \\
12 \div 1 = 12 \\
12 \div .1 = 120 \\
12 \div .01 = 1200 \\
12 \div .001 = 12000 \\
12 \div .0001 = 120000 \\
\quad \vdots \qquad\quad \vdots \\
12 \div -.0001 = -120000 \\
12 \div -.001 = -12000 \\
12 \div -.01 = -1200 \\
12 \div -.1 = -120 \\
12 \div -1 = -12 \\
12 \div -2 = -6 \\
12 \div -3 = -4
\end{array}
$$

Here the dividend is constant. The divisor varying continuously, suppose, changes sign in passing through zero (absolute) and at the same time the quotient changes sign in passing through infinity.

The following definition of division may assist in reaching a conclusion : Division is the process of finding how many times a number may be subtracted from another without changing the sign of the remainder.

Apply this definition thus : How many times may zero (absolute) be subtracted from 12 without changing the sign of the remainder.

The answer is, an infinity of infinities, rather than zero.

MILTON L. COMSTOCK.

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Upon some of the points about which I shall disagree with Dr. Lilley he can quote in his favor some of the most brilliant mathematicians that the world has produced. Nevertheless I shall endeavor to show that they and he have failed to take a common sense view of the subject. Upon one point I think I am safe in saying that the Doctor's position is unique. The source of his errors lies, in my opinion, in his conception of infinity and zero.

Concerning the former he says : "If 12/⦶=infinity or *the largest possible number*," etc.

From this I can not but infer that he thinks infinity is a constant and that that constant is the largest possible number. He says "12/⦶=⊖, where ⊖ represents no number of times." Again we infer that he believes that while ⊖ represents *no number of times*, ∞ must represent *some number of times*.

He uses too many zeros. He has ⦶=absolute zero, ⦶=*no number of times*, and ◎=an infinitesimal. He refers to the latter zero as follows : It is a consequence of confounding the 0 arising from dividing *a* by infinity, with the absolute zero, that so much confusion has arisen."

He has the authority of Davies and Peck's Mathematical Dictionary for this statement, but this does not make it true. Nothing could be more confusing to the average man of common sense than the Doctor's three zeros.

I have no use for more than one. My mind is perfectly clear as to what that is but it is not so clear as to what ∞ is. It is much easier to tell what ∞ is not than what it is.

If we suppose $a/h=N$, where h is a very small positive quantity, then N is a very large one. As h grows smaller and smaller, N grows larger and larger, but N will *not* become infinite *so long as h has the smallest shadow of value.* So long as h has the slightest value we can form some conception of the value of N. It is only when h becomes equal to 0 that N suddenly swings clear out of our powers of conception. It is then, and then only, that it becomes infinite.

I must dissent from even so great a mathematician as Professor De Morgan when he said that he dated his first clear conception of mathematical infinity from the time when he rejected the relation $a/0=\infty$.

The very fact that he had a clear conception of what he called infinity proved that it was not the real infinity.

I have no criticism to make on Dr. Lilley's disposition of 0/0. I would have liked it better if he had added Art. 175 of his Higher Algebra, which reads: "The symbol 0/0 does not always mean *indetermination.* It is often the result of a particular condition which makes a factor, common to both terms of a fraction become zero. Thus," etc. Here follows the well known illustration by using $\frac{a^2-x^2}{a-x}=a+x$.

He does not find it necessary to introduce the infinitesimal to prove that the expression equals $2a$ when $x=a$, as do many writers on the differential calculus when discussing the expression $\frac{y_1-y}{x_1-x}$. In this he is right, for if $a-x$ were an infinitesimal the value of the fraction would differ from $2a$ by an infinitesimal, and *an equation that differs from the truth by an infinitesimal is not true at all.*

HENRY HEATON.

Atlantic, Iowa.

We see no place for confusion in the use of the symbols 0 and ∞, and, therefore, of course, no necessity of introducing new symbols to avoid confusion. If 0 is a symbol used to denote the absence of quantity, and ∞ to denote a quantity larger than any assignable quantity however large, then all operations with these symbols are meaningless. For example, $5\div\infty$, $0\div5$, $5\div0$, $0\div0$, $0\times\infty$, etc., are impossible operations. Standing apart from conditions imposed upon quantities from which these symbols arise by certain limitations, they have no meaning whatever. Hence, when these symbols do arise in mathematical investigations, they must be interpreted in conformity to fundamental principles and conceptions. When we say that $5\div\infty=0$, we mean that the limit of $5\div$ a quantity which increases indefinitely$=0$, concisely expressed thus $\lim_{h\doteq\infty}\left[\frac{5}{h}\right]=0$.

This is an absolutely accurate statement. 0 is the absolute zero and not an *infinitesimal.* In like manner $5 \div 0 = \infty$ is an abbreviated and inaccurate expression for the following: 5 divided by a quantity which decreases indefinitely gives a quotient larger than any quantity however large, or briefly and *accurately* thus $\lim_{e \doteq 0} \left[\frac{5}{e}\right] = \infty$.

Discussion on a subject of this sort is trivial, but if it results in giving clearer notions of the use of 0 and ∞, a good work will have been done.

B. F. F.

BOOKS AND PERIODICALS.

Plane and Solid Analytical Geometry. By Frederick H. Bailey, A. M. (Harvard), and Frederick S. Woods, Ph. D. (Göttengen), Assistant Professors of Mathematics in the Massachusetts Institute of Technology. 8vo. Cloth, 371 pages. Boston and Chicago : Ginn & Co.

Besides the usual subjects treated in the ordinary text-books of Analytical Geometry, the following additional ones are treated with sufficient fullness to give a student a fair knowledge of them, viz: Radical Axis, and Properties of Pole and Polars. More attention should be given to these subjects in the future by the ordinary student. Besides deriving the equations of the conics in the usual way, the authors have also derived the equations by passing a plane through a right circular cone, thus emphasizing the relation of the geometrical to the analytical method of treatment. About seventy pages are given to the treatment of Solid Analytical Geometry. The treatment here is clear and concise, affording the student an excellent introduction to this important subject. B. F. F.

Famous Problems of Elementary Geometry.—The Duplication of the Cube ; The Trisection of an Angle ; and The Quadrature of the Circle. Authorized translation Vorträge Ueber Ausgewälte Fragen der Elementargeometrie Ausgearbeitet von F. Tägert. By Wooster Woodruff Beman, Professor of Mathematics in the University of Michigan, and David Eugene Smith, Professor of Mathematics in the Michigan State Normal College. 8vo. Cloth, 80 pages. Price, 55 cents. Boston and Chicago : Ginn & Co.

This book deals with the possibility of elementary geometric constructions in general, the nature of transcendental numbers, and with the transcendence of e and π. While no knowledge of the calculus is needed to read this book, the calculus not being employed in any of the discussions, yet a fair knowledge of the theory of equations and series is absolutely necessary to make it easy reading. The translators deserve the thanks of students and teachers of mathematics, and for putting out books of such scientific value at a very reasonable price, the publishers should receive encouragement by a large sale of this book.

B. F. F.

Popular Scientific Lectures. By Ernst Mach, formerly Professor of Physics in the University of Prague, now Professor of the History and Theory of Inductive Science in the University of Vienna. Translated by Thomas J. McCormack. Second Edition, Revised and Enlarged. 8vo. Cloth, 382 pages. Price, $1.00. Chicago : The Open Court Publishing Co.